Modeling the Dynamics of EU Economic Sentiment Indicators: An Interaction-Based Approach*

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Abstract

This paper estimates a simple univariate model of expectation or opinion formation in continuous time adapting a "canonical" stochastic model of collective opinion dynamics (Weidlich and Haag, 1983; Lux, 1995, 2007). This framework is applied to a selected data set on survey-based expectations from the rich EU business and consumer survey database for twelve European countries. The model parameters are estimated through maximum likelihood and numerical solution of the transient probability density functions for the resulting stochastic process. The model's success is assessed with respect to its out-of-sample forecasting performance relative to univariate time series models of the ARMA(p,q) and ARFIMA(p,d,q) varieties. These tests speak for a slight superiority of the canonical opinion dynamics model over the alternatives in the majority of cases.

Keywords: Expectation formation, Fokker-Planck equation, forecasting.

1. Introduction

It is widely believed that expectations play a major role in determining macroeconomic outcomes. Unfortunately, there is no consensus about the appropriate modeling of *expectation formation*. Many theories and approaches have been suggested in the literature to formalize this important ingredient of economic models. Over the last decades, the rational expectations hypothesis has become the dominant paradigm of modern macroeconomic theory and survey data have been used to test for rational expectations of respondents, mostly with not too much support for rationality[1]. However, little has been done to test *alternative* theories of expectation formation using the vast amount of survey data on empirical expectations that are regularly published by private and academic institutes or governments in most developed countries.

Branch (2004), Carroll (2003) and Roberts (1998) are some of the rare examples that consider alternative theories of expectation formation that do not impose homogeneous rational expectations. While there is a scarcity of theoretical models of boundedly rational expectation formation, extant empirical research on survey-based

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[1]e.g., Acemoglu and Scott, 1994; Delorme et al., 2001, and the survey by Nardo, 2003.

expectations is quite rich. As an obvious research question a large number of papers investigates the predictive capacity of survey data for consumer spending or output (e.g., Lemmens et al., 2005; Taylor and McNabb, 2007; Gelper er al., 2007) or seeks for determinants of sentiment in macroeconomic or political data (Vuchelen, 1995) or even compact measures of the generally optimistic or pessimistic disposition of a society (e.g. Zullow, 1991 who uses indices of positive and negative moods in pop songs and news articles). The later studies are close to our approach in so far as they presume some kind of propagation of a dominant mood via direct or indirect interaction. Popular culture and mass media might, then, both reflect and reinforce overall mass-psychological trends in a society. Our goal here is to contribute to such a behavioral theory of sentiment formation by moving from a purely statistical analysis to an explicit modeling of the interaction effects in consumer or business surveys. Such an attempt at modeling and testing alternative hypotheses of opinion and expectation formation is a relatively recent strand of literature. We follow closely the recent work by Lux (2007) and Franke (2007) who both estimate (with different econometric techniques) the parameters of a “canonical” opinion dynamic model introduced below for a particular German business survey.

This study provides an empirical assessment of this opinion formation model on the base of social interaction using the rich EU business and consumer survey database for twelve European countries as collected and released by the European Commission Directorate-General For Economic and Financial Affairs [henceforth, the Commission]. In particular, a simple univariate model of opinion or expectation formation in continuous time is postulated in the spirit of Weidlich and Haag (1983). Following the methodology of Lux (2007), based on previous contributions by Poulsen (1999) and Hurn *et al.* (2006), the model parameters are then estimated via approximate maximum likelihood. Since no closed-form solution of the transient density of this model is available, our ML algorithm will be based on the numerical solution of the relevant Fokker-Planck equation (the partial difference equation governing the dynamics of the pdf) using a finite difference approximation. The model's goodness-of-fit is checked with respect to its out-of-sample forecasting performance relative to standard univariate time series models of the ARMA(p,q) and ARFIMA(p,d,q) varieties. The results of these tests speak for the moderate superiority of the canonical continuous-time model over the alternatives, ARFIMA (10 successful cases out of 36), and ARMA (2/36), i.e. in approximately 67% of cases.

The paper proceeds as follows. Section 2 introduces briefly the content of the survey data under investigation. Section 3 sketches the theoretical framework suggested to model such data. Section 4 provides the empirical analysis and checks the goodness-of-fit of the model against pure time series models. Section 5 considers briefly potential extensions of the canonical model. Section 6 concludes and indicates further directions of research under the framework of this paper.

2. Overview of the EU Business and Consumer Survey Data

National institutes in the EU Member States and candidate countries regularly conduct business and consumer surveys on behalf of the Joint Harmonised EU Programme of Business and Consumer Surveys [henceforth, BCS programme]. The collected data are published in the media by the Commission and are freely available.

The way in which answers obtained from the surveys are compiled and released is worth mentioning since it is the aggregate information that is used in our estimation. The responses are aggregated in the form of "balances" or diffusion indices. Balances are constructed as the difference between the percentages of respondents giving positive and negative replies. "Neutral" answers are ignored. For example, if among the total number of N^* respondents[2] (for some specific question) "positive" ("negative") answers are given by N^+ (N^-) individuals, then the *balance*, *B*, is computed as follows

$$B = (N^+ - N^-) / N^* .$$

The *balance series*[3] constitute the major part of the output data of the BCS programme. These series are further used to build *composite indicators.* From the vast amount of the available EU survey data the particular questions chosen for the analysis in this paper are the following that relate to future expectations:

- Industry Survey, Q5: *How do you expect your production to develop over the next 3 months? It will...*
- Construction Survey, Q4: *How do you expect your firm's total employment to change over the next 3 months? It will...*
- Retail Trade Survey, Q4: *How do you expect your business activity (sales) to change over the next 3 months? It (They) will...*

  $+$ *improve (increase)* $=$ *remain unchanged* $-$ *deteriorate (decrease)*
- Consumer Survey, Q4: *How do you expect the general economic situation in this country to develop over the next 3 months? It will...*[4]

$++$	*get a lot better*	$=$	*stay the same*	$-$	*get a little worse*
$+$	*get a little better*	*DK*	*don't know*	$--$	*get a lot worse*

The questions that the next sections attempt to answer are: How could we model expectation formation of agents faced with the above questions? Do agents independently form expectations or can we identify some sort of social interaction between respondents? Can we predict future expectations? How good are our forecasts? What could be done in order to improve predictions?

3. A Framework for Collective Opinion Formation

As a model of expectation formation we adopt a stochastic framework along the lines of Weidlich and Haag (1983) and Lux (1995). The model is stylized and is based on a set of mass-statistical regularities governing respondents' switches between two possible opinions. Assuming that (A1) the total number of respondents is constant and, without loss of generality, is given by $N^* = 2N$, and that (A2) the respondents are allowed to have only two relevant opinions or expectations, denoted by $+$ and $-$, let n_t^+ and n_t^- represent the *numbers* of agents holding at time t the positive and

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[2]Note that N^* includes the number of "neutral" agents, $N^{\sim}$, i.e. $N^* = N^+ + N^- + N^{\sim}$.

[3]Balance series are usually referred to as "opinion index"', "climate index"', or "diffusion" in the literature.

[4]Note that in the case of the last question the balance is calculated as

$$B = [(N^{++} + 1/2N^+) - (1/2N^- + N^{--})] / N^*$$

with the intuitive notation of N^{++} (N^{--}) being the number of "very optimistic" ("very pessimistic") respondents.

negative expectations, respectively. Define the *configuration*, n_t, as follows $n_t := (n_t^+ - n_t^-)/2$, $-N \le n_t \le N$, and introduce the notion of *aggregate* or *average expectations* as the ratio

$$x_t := n_t / N,$$

with $-1 \le x_t \le 1$. Since all agents have equal weight in the population, we can interpret the state $x_t = 0$ as representing the balance between overall optimism and pessimism, with the states $x_t > 0$ and $x_t < 0$ describing the cases of optimistic and pessimistic *majorities*, respectively. This opinion index is our proxy for the balance series (for more details see the full manuscript). As time passes individual agents may change their opinions about the relevant questions. Thus they might switch from being optimistic to becoming pessimistic and *vice versa*.

Utilizing the *Master equation*[5] formalism of stochastic calculus it is possible to show that the evolution of x_t can be described by the following stochastic differential equation [SDE]

$$dx_t = A(x_t;\theta)dt + \sqrt{D(x_t;\theta)}dW_t,$$

where W_t denotes the standard Wiener process, and

$$A(x;\theta) = v(1-x)e^{\alpha_0+\alpha_1 x} - v(1+x)e^{-\alpha_0-\alpha_1 x},$$
$$D(x;\theta) = [v(1-x)e^{\alpha_0+\alpha_1 x} + v(1+x)e^{-\alpha_0-\alpha_1 x}]/N,$$

where θ denotes the parameter vector, $\theta = (v, \alpha_0, \alpha_1)'$.

Since this is a stochastic model for the aggregate behavior of our pool of respondents, a characterization of the outcome of this process requires to track the temporal development of the density $P(x;t)$. Conditional on some initial value, the transient density follows the *Fokker-Planck equation*[6]

$$\frac{\partial P(x;t)}{\partial t} = -\frac{\partial}{\partial x}\{A(x)P(x;t)\} + \frac{1}{2}\frac{\partial^2}{\partial x^2}\{D(x)P(x;t)\}.$$

Unfortunately, with the highly non-linear drift and diffusion terms of our system, no closed-form analytical solution to this equation is available. We will, therefore, rely on numerical approximations of the Fokker-Planck equation in our empirical application. However, it is easier to derive the equilibrium properties of this system. The stationary distribution can be obtained by setting the left hand side of the Fokker-Planck eq. equal to zero, $\partial P(x;t)/\partial t = 0$. We do not reproduce the closed-form solution of the stationary density here, but summarize its important properties[7]:

1. For $\alpha_1 \le 1$, the stationary distribution of the process x_t is characterized by a unique maximum (mode). If $\alpha_0 = 0$, this maximum is located at $x^* = 0$. It shifts to the right (left) for $\alpha_0 > 0$ (<0).
2. For $\alpha_1 > 1$ and α_0 not too large, the stationary distribution has two maxima (two modes) $x_+ > 0$ and $x_- < 0$. If $\alpha_0 = 0$, the bimodal distribution is

[5]The Master equation represents the general and exact system of equations tracking the flow of probabilities between states, see Weidlich and Haag (1983) or Van Kampen (2007).
[6]See Weidlich and Haag (1983), Lux (1997, 2007), Gardiner (2004), and Van Kampen (2007) for more details.
[7]cf. Weidlich and Haag (1983), Lux (2007).

symmetric around 0. It becomes asymmetric if $\alpha_0 \neq 0$ with right-hand (left-hand) skewness and more concentration of probability mass in the right (left) maximum if $\alpha_0 > 0$ (<0) holds.

3. If $|\alpha_0|$ becomes very large, the smaller mode vanishes and the stationary distribution becomes uni-modal again. This happens if $|\alpha_0|$ increases beyond the bifurcation value α_0^* which can be computed. For more details we refer to Lux (2007) and Ghonghadze and Lux (2008).

4. Empirical Results

The main objective of this paper is to estimate the parameters of our behavioral model for the selected balance series and assess the performance of this model as a hypothesized data-generating process for the BCS sentiment data. Since the EU Business and Consumer Survey Data is huge we have chosen only the four questions described in Section II for only those twelve countries for which the series were available from 1985. Thus, for each single question-country pair we have a sample of 276 monthly observations. In order to test the forecasting power of the model we have chosen the first 192 observations as our in-sample, covering the 16 year period 01.1985 -- 12.2000. The number of out-of-sample observations is 84, covering the next 7 year period 01.2001 – 12.2007. These data series are seasonally adjusted by the provider.

Previous experience indicates the need to consider different parameterizations of the basic univariate interaction-based model (see Lux (2007)). Therefore the following set of four models has been estimated:

M1: The parameter vector to be estimated is $\theta = (v, \alpha_0, \alpha_1)'$, with the number of respondents fixed at the "official" number given in the documentation of the BCS programs[8]. Model 1 is exactly the full model specified above and will henceforth be referred to as the "canonical model".

M2: N is fixed and is given as in Model 1. The parameter vector to be estimated is $\theta = (v, \alpha_1)'$. Here we neglect the bias parameter α_0. The reason is that for relatively weak interaction (α_1 small), approximate colinearity between α_0 and α_1 could impede our estimation.

M3: Under Model 3 N is no longer fixed. The parameter vector to be estimated is thus $\theta = (v, \alpha_0, \alpha_1, N)'$. Here we let the in-sample data provide the information about the implied "effective" number of respondents[9].

M4: As in Model 3, N is also not fixed here. The parameter vector to be estimated is $\theta = (v, \alpha_1, N)'$. We neglect the effect of the bias parameter α_0.

The total number of models that we estimate thus amounts to 144. Below in sec. 5 we will also consider a slight extension of our models M1 to M4.

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[8]This information can be obtained from the EU Business and Consumer Surveys database. It actually varies widely across countries and sections.

[9]The idea is that despite the inclusion of a social interaction term our model might not capture all correlation between respondents. For example, there might be groups that always switch simultaneously which would, indeed, reduce the number of effectively independent agents. Of course, the officially reported number should be an upper boundary to the "effective" number.

4.1. Estimation

Note that we use discrete observations in order to estimate the parameters of a continuous-time process of opinion formation. For a sample of observations $x_0,...,x_T$ we can estimate these parameters most efficiently via maximum likelihood. The log likelihood of our sample of observations amounts to

$$\log P_0(x_0 \mid \theta) + \sum_{s=0}^{T-1} \log P(x_{s+1} \mid x_s, \theta) \, .$$

The conditional probabilities $P(x_{s+1} \mid x_s, \theta)$ can be obtained by numerical iteration of the Fokker-Planck equation over a unit time interval taking x_s as the initial condition. Since we do not have a previous observation for x_0, we have to use the unconditional probability $P_0(x_0 \mid \theta)$ to evaluate this component (however since its influence is negligible, we will simply skip this observation in our empirical applications).

For the numerical approximation of the Fokker-Planck equation we follow the methodology developed by Lux (2007) and suggested earlier by Poulsen (1999) and Hurn et al. (2006) in a different context. First, the Fokker-Planck equation is solved numerically via a Crank-Nicolson finite difference scheme. Then, the log-likelihood function is evaluated for the 192 in-sample observations and is numerically maximized with respect to the unknown parameters. More details on the numerical aspects can be found in Lux (2007). Computations have been performed in GAUSS. The results are summarized in Tables 1 – 12 in the companion manuscript (see Ghonghadze and Lux, 2008).

4.2. Goodness-of-fit

The goodness-of-fit of all four models is checked with respect to their out-of-sample forecasting performance relative to a benchmark. In particular, one month out-of-sample forecasts are constructed for all models and two types of forecasting errors are computed: *root mean-squared errors* [RMSE] and *absolute mean errors* [AME]. The same quantities are calculated for univariate time series models such as ARMA(p,q) and ARFIMA (p,d,q), which serve as our benchmarks.

4.3. Forecasting

This paper considers two different one-month-ahead forecasts for the models M1--M4: *expected value* and *nearest maximum* of the predictive density function. The term “expected” stands for the expected value of the opinion index x_{t+1} at one-month horizon conditional on its value one month earlier, x_t. The needed predictive density is again obtained via numerical solution of the Fokker-Planck equation with the previously estimated parameter vector $\hat{\theta}$. A similar procedure applies to the computation of the “nearest” forecast, which is the nearest maximum of the predictive density, and therefore represents the most likely mode of the opinion index at some future date. These “expected” and “nearest” forecasts are calculated for the out-of-the-sample data of respective balance series.

In order to set benchmarks, we have also estimated the best ARMA(p,q) and ARFIMA(p,d,q) in-sample. For ARMA we have set $p,q \leq 1$ $p,q \leq 5$, for ARFIMA,

$p, q \leq 1$ (as the longer lags should be captured by the parameter of fractional differentiation). From the range of the ARMA and ARFIMA models within this set, the one that minimizes the Akaike information criterion is chosen for forecasting. Then, based on the fitted models, out-of-sample one-month-ahead forecasts have been computed.

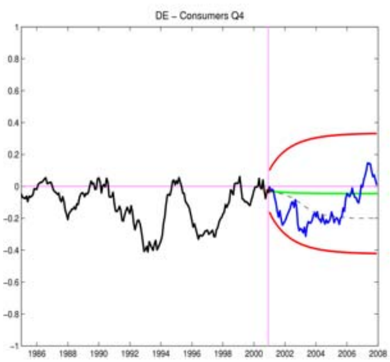
Empirical results show the following regularities:

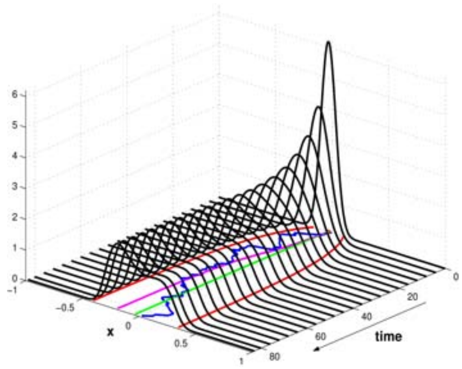
1. ARMA forecasting accuracy is usually outperformed by both, the predictive power of ARFIMA and that of models M1--M4. The exceptions from this pattern are the cases of Irish and French industries (see Tables 4 and 6 in the manuscript). For example, both Model 1 and the ARFIMA model outperformed ARMA with respect to RMSE and AMSE in 94.4% of all cases. In 24 cases out of a total of 36 the canonical continuous-time model was slightly superior to the ARFIMA (which dominated in 10 out of 36 cases) and ARMA (2 out of 36) models, i.e. in approximately 67% of cases. For M2 to M4 the results are very similar.
2. Considering the full range of our interaction-based models M1--M4 we find better fits of at least one specification with respect to RMSE and AMSE than the best ARFIMA performance in the majority of cases which correspond to 75% of our experiments (27 cases out of 36).
3. Predictive accuracy within the family of interaction-based models is usually increasing and only sometimes slightly decreasing when going from M1 to M4, whereas the estimated values for corresponding log-likelihoods do mostly not improve essentially. This is surprising since allowing for *N*, the “effective” number of participants as a free parameter, provided for a crucial improvement of goodness-of-fit in the case of a German sentiment index (Lux, 2007).
4. The Diebold-Mariano test could not reject the null hypothesis of equal predictive accuracy at the 5% level between (a) the expected value and ARFIMA forecasts in 90.3% of cases, (b) the nearest value and ARFIMA forecasts in 88.9% cases, and (c) the expected value and the nearest value forecasts in 88.2% of cases, when the total number of 144 experiments is taken into account.
5. For the parameter of the opinion model, we typically find the crucial entry for the intensity of interaction, α_1, to be in the close vicinity of its bifurcation value 1 (at which system behavior switches from uni-modal to bi-modal) for Model 1. However, allowing for endogeneity of *N*, this value mostly turns out to be lower. Similar finding have been reported in Lux (2008). It appears that there is a trade-off between the number of independent agents and their interaction intensity: If we insist on *N* in accordance with the official numbers, the model can only reproduce the fluctuations of the index with α_1 close to its crucial value. If we allow for a lower number of “effectively independent” agents, lower interaction intensity will be sufficient in the best fit to our model[10].

An Example. In order to illustrate what the potential added explanatory power of our opinion model could be, we exhibit some more details in the case of German

[10]Note that using the large official numbers of respondents would lead to very low predicted volatility due to the law of large numbers. This can to a certain degree be overcome by high sensitivity of the system to changes. This is what characterizes the vicinity of while moving away from this benchmark in both directions leads to more persistent macroscopic dynamics.

consumers. This is particularly interesting because the fitted canonical model displays the possibilities for *phase transitions*. In particular, both M1 and M2 have α_1 parameters higher than unity. This setting corresponds to the bi-modal equilibrium distribution of consumer opinions, switching from optimistic to pessimistic long-run equilibria and *vice versa*. In Figure 2 below we have graphed in- and out-of-sample observations for the series. We have also superimposed two standard deviation bounds on the evolution of the predictive density conditional on the very last in-sample observation, $x_0 = -0.03$. As parameters we have chosen the simple averages of the two models, M1 and M2: $v = 0.5475$, $\alpha_0 = -0.0006$, $\alpha_1 = 1.0109$, $N = 1000$. As can be seen from the graph, we are able to track the global maximum of the predictive density (the dashed black curve). The evolution of the mean is given by the green line and it stays closely in the neighborhood of the initial condition. The evolution of the global maximum, on the contrary, diverges from the mean downwards a strongly negative configuration. Why is the global maximum not capable of pulling the mean with it? What force keeps the dynamics of the mean almost unchanged? The answers to these questions are provided by Figure 3. It displays the complete evolution of the predictive density. We see how a second local maximum develops in the positive quadrant. This is the very reason for the observed dynamics of the mean that roughly corresponds to the average between both modes.





Figures 2 and 3: The evolution of the moments of the predictive density conditional on the last in-sample observation, $x_0 = -0.03$.

5. An Extension

The framework of the interaction-based model for expectation formation can be easily extended to incorporate the effects of important exogenous macroeconomic variables. In order to allow for additional determinants in the opinion process, one could simply expand the forcing function, $U(x_t) = \alpha_0 + \alpha_1 x_t + \alpha_2 Y_t$, where α_2 is an *m*-dimensional vector of coefficients and Y represents the *m*-dimensional vector of relevant macro variables. The case $Y_t = \Delta x_t$ corresponds to the so-called "momentum effects", where Δx_t denotes the difference between the time *t* and the time *(t-1)* observations of the index which stays fixed over the time interval $[t, t+1)$, see the study of Ghonghadze and Lux (2008). The idea is that respondents might react not only to the net influence of their environment but also be particularly sensitive to changes of the index itself.

6. Conclusions

This paper has explored the explanatory and predictive power of a non-rational model of opinion formation among interacting agents for European BCS data. Applying the canonical model of opinion formation by Weidlich and Haag (1983) to four selected indices across 12 core countries of the European Union, we found the following:

1. In contrast to our pilot application of the present estimation methodology in Lux (2007), different specifications of the model make little difference to its in-sample and out-of-sample fit. In particular, we found little improvement through adding a "momentum"effect to the opinion dynamics.
2. With respect to its forecasting performance out-of-sample, the endogenous opinion model typically did better than an ARMA model. Compared to the more persistent ARFIMA class, predictive power was mostly not significantly different for single series (as judged by the Diebold-Mariano test). However, for the cross-section of data as a whole, we find a dominance of the opinion model in about two thirds of all cases (although its advantage over ARFIMA might be small).

It is worthwhile to note that it is not really clear whether we could expect more predictive power if this model were the "true" data generating process. On the one hand, the model output is characterized by stochastic switches between two maxima of its probability density in the case of strong interaction. Although our model could help in understanding such transitions between prevailing optimism and pessimism, the stochasticity of these swings would prevent successful prediction of changes of the public's mood. On the other hand, if interaction is relatively weak ($\alpha_1 < 1$), the persistency of the stochastic ARFIMA model might be a good approximation built-in to the behavioral persistency of the opinion model[11]. Both aspects need to be explored in order to get an idea of the potential forecasting performance of such models.

A certain deficit of our present approach is the uni-variate nature of our models. Of course, the opinion dynamics will not be decoupled from other economic data and might be influenced by exogenous news about economic and possibly political conditions. Such factors could enter the formalization of transition rates (as we did in sec. 5)[12] or we could combine our opinion dynamics with additional dynamic components formalizing the time development of, for example, GDP, interest rates, etc. One would, then, hope to disentangle the influence of objective factors from intrinsic propagation of moods among the population of respondents. This daunting task is left for our future research.

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[11]Alfarano and Lux (2007) demonstrate that a closely related model mimics the long-term dependency, the defining feature of ARFIMA models. Lux (2008) shows that both a behavioral opinion model and a parsimonious diffusion process provide nearly equivalent fits to a financial sentiment index.

[12]Lux (2007) considered various macroeconomic factors in the analysis of a German business climate index but found surprisingly little value added compared to the "canonical" model.

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