Measurement and Inference on Cross Section and Spatial Interactions

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Abstract

Traditionally, research has been devoted almost exclusively to estimation of underlying structural models without adequate attention to the drivers of diffusion and interaction across cross section and spatial units. We review some new methodologies in this emerging area and demonstrate their use in measurement and inferences on cross section and spatial interactions. Limitations and potential enhancements of the existing methods are discussed, and several directions for new research are highlighted.

Keywords: Cross Sectional and Spatial Dependence; Spatial Weights Matrix; Interactions and Diffusion.

1. Introduction

Many studies in applied statistics and econometrics concern situations where there is substantial cross section dependence. The usual approach to the representation of such spatial dependence is to define a spatial weights matrix, which represents a theoretical and a priori understanding of the nature of spatial interdependence between different geographical regions or, more generally, between different economic agents. These spatial weights represent patterns of interaction and diffusion, and thereby provide a meaningful and easily interpretable representation of spatial interaction (spatial autocorrelation) in spatial dependence models. In combination with spatial structure (spatial heterogeneity), they offer a useful framework for studying cross-sectional dependence (see, for example, Anselin, 1988, 1999). The spatial weights are usually interpreted as functions of relevant measures of economic or geographic distance (Anselin, 1988, 2002). The distance between two agents reflects their proximity with respect to unobservables, so that the joint distribution of random variables at a set of points can be represented as a function of the economic distances between them.

The choice of appropriate spatial weights is a central component of spatial models as it imposes a priori a structure of spatial dependence, which may or may not correspond to reality. Further, the accuracy of these measures affects profoundly the estimation of spatial dependence models (Anselin, 2002; Fingleton, 2003). Indeed, a challenge for empirical analysis and structural explanations is that interaction between observation units often cannot be either precisely measured or in other ways explained by observed measures of distance.

This is particularly true since, in many applications, there are multiple possible choices and substantial uncertainty regarding the appropriate choice of distance measures. However, while existing literature contains an implicit acknowledgment of these issues, most empirical studies treat spatial (and sometimes spatio-temporal) dependence in a superficial manner assuming inflexible diffusion processes in terms of known, fixed and arbitrary spatial weights matrices (Giacomini and Granger, 2004). The problem of choosing spatial weights becomes a key issue in many applications.

In this paper, we review the current literature on methods that take a nonparametric view on the nature and strength of spatial diffusion and cross section interaction. As a new departure from the literature, this growing volume of research is developing methods for estimating spatial weights (or interactions) that are consistent with an observed pattern of spatial (or cross sectional) dependence. Once these interactions have been estimated they can be subjected to interpretation in order to identify the true nature of spatial dependence, representing a significant departure from the usual practice of assuming a priori the nature of spatial interactions. The methods are illustrated with examples, and important open research questions are highlighted.

Consider, for example, an application to diffusion in housing demand or prices across housing markets in England. It is commonly believed that most housing shocks originate from London, though how these shocks then propagate through space is not well understood. In this context, questions like the following are natural: which is closer to London – Hertfordshire, Buckinghamshire or Birmingham? If geographic distance is the relevant criteria, Hertfordshire and Buckinghamshire are much closer to London than Birmingham. However, if the criterion is sociocultural distance, Birmingham may be closer. Then, which distance measure is more appropriate for spatial dynamics in housing markets? Previous research (Bhattacharjee and Jensen-Butler, 2005), discussed later, suggests that spatial spillover from excess housing demand in London are significantly higher for the South East (containing Buckinghamshire) and the West Midlands (containing Birmingham) than they are for the East of England (containing Hertfordshire). Thus, while geographic distance may be a useful measure to understand spatial diffusion between certain regions (like London and the South East), it is certainly not appropriate for some other regions (like London and West Midlands or London and the East of England). This situation highlights typical challenges faced by researchers studying spatial diffusion in housing demand and price signals – there are many possible drivers of the diffusion process, none of which explains diffusion in all regions.

The paper is organised as follows. In Section 2, we follow Bhattacharjee and Jensen-Butler (2005) and describe estimation of the spatial weights matrix in a spatial error model. We emphasize that estimation of spatial weights consistent with an estimated pattern of spatial autocorrelations is a partially identified problem, and therefore structural constaints are required for precise estimation; symmetry of the spatial weights matrix constitutes such a valid set of identifying restrictions. Section 3, based on Bhattacharjee and Holly (2008a), considers estimation and inference on interactions under moment restrictions which explicitly exploit the spatio-temporal nature of panel data on economic agents. While the previous methods consider interactions that are deeply structural, Section 4 describes methods (Holly et al., 2008) that allow for spatial effects that may be driven by unobserved common factors.

Following this, we discuss issues relating to sampling of spatial units. We argue that the methods discussed can be used to suggest appropriate sampling plans (Section 5). However equally importantly, the methods can deal with endogeneity in the selection of sampling units, and that a selection should be retained for a period of time to allow for appropriate elicitation of spatial effects in a panel setting. Finally, Section 6 concludes, highlighting strengths and weaknesses of the discussed methods, as well as areas of new applications and research.

2. Estimation of spatial weights in a spatial error model

Bhattacharjee and Jensen-Butler (2005) consider estimation of a spatial weights matrix in a spatial error model with spatial autoregressive errors. They consider a setting where a given set of cross section units have fixed but unrestricted interactions; these interactions are inherently structural in that they are related to an underlying structural economic model.

Specifically, Bhattacharjee and Jensen-Butler (2005) consider a structural spatial model of regional housing demand (*D*). Demand within a region is explained by the effects of explanatory variables (***X***) and diffusion of excess demand from other regions. The spatial externalities in the form of demand diffusion are driven by the completely unspecified pattern of spatial weights (given by the spatial weights matrix ***W***) and the strength of spatial spillovers in each region described by spatial autocorrelation parameters ρ_K. The model and its reduced form are as follows:

$$\begin{aligned} \underline{D}_t &= \mathbf{X}_t.\underline{\beta} + \underline{u}_t, \qquad t = 1, \ldots, T \\ \underline{u}_t &= \mathbf{R}.\mathbf{W}.\underline{u}_t + \underline{\varepsilon}_t, \\ \Rightarrow \underline{D}_t &= \mathbf{X}_t.\underline{\beta} + (\mathbf{I} - \mathbf{R}.\mathbf{W})^{-1}.\underline{\varepsilon}_t, \end{aligned}$$

where there are T time periods ($t = 1, \ldots, T$) and K regions ($k = 1, \ldots, K$), D_t is the $K \times 1$ vector of regional demand in period t, $\boldsymbol{W}$ is an unknown spatial weights matrix of dimension $K \times K$, $\boldsymbol{R} = diag\,(\rho_1, \rho_2, \ldots, \rho_K)$ is a $K{\times}K$ diagonal matrix containing the (possibly heterogeneous) spatial autoregression parameters for each region, and ε_t is the $K \times 1$ vector of independent but possibly heteroscedastic spatial errors.

Bhattacharjee and Jensen-Butler (2005) show that without any structural constraints on the spatial weights matrix,[1] the estimation problem is only partially identified, up to an orthogonal transformation of interactions. Symmetry of the spatial weights matrix constitutes one set of valid identifying restrictions. Under such assumptions, they describe inference methods and an algorithm for estimation of the unknown spatial weights matrix.

Application of the methods for understanding spatial diffusion in housing demand across government office regions in England and Wales provide important and interesting inferences. The topological map implied by the estimated spatial weights (Figure 1) demonstrates that, while contiguity and geographic distance provide some understanding of the strength of inter-region interactions, other factors such as socio-cultural distances and transport infrastructure may also drive the spatial diffusion in housing demand. Further, some significant negative spatial interactions (not allowed

[1] Other than the condition that $(\boldsymbol{I} - \boldsymbol{R}.\boldsymbol{W})$ is nonsingular which is required for identification in the reduced form.

in typical spatial econometric applications based on geographical distance) suggest alternative markets and asynchronous housing cycles.

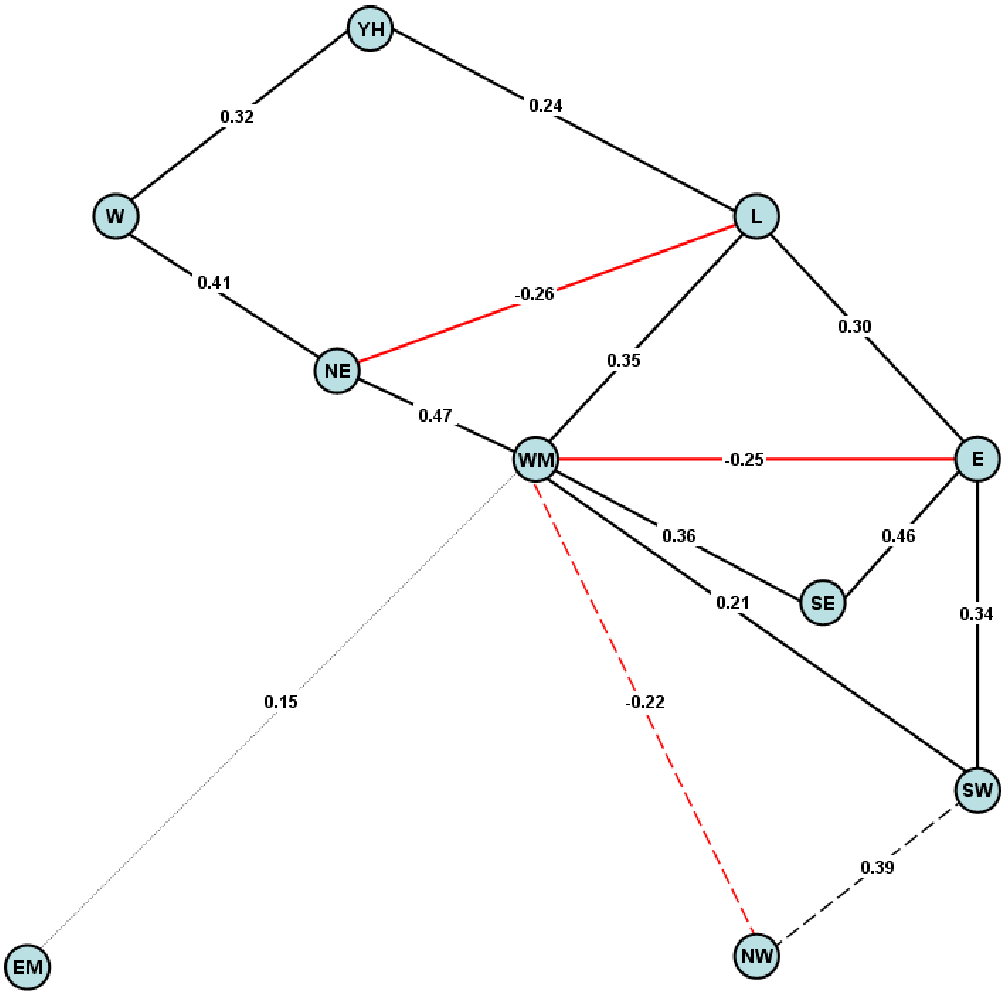

Figure 1: Topological Map of England and Wales based on spatial weights[2,3]

However, the identifying restrictions of symmetry may be too strong in some applications. Bhattacharjee and Holly (2008b) apply similar methods to committee decision making under a different set of identifying structural constraints.

3. GMM based inferences on endogenous interactions

As discussed above, identifying structural restrictions in Bhattacharjee and Jensen-Butler (2005) can be quite restrictive in many applications. Besides, standard errors in the above method have to be estimated by a bootstrap procedure which can be cumbersome. Bhattacharjee and Holly (2008a) develop an alternative GMM based methodology for estimating spatial or interaction weights matrices which are unrestricted except for the validity of the included instruments and other moment conditions.

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[2] E: East of England; EM: East Midlands; L: Greater London; NE: North East; NW: NorthWest; SE: South East; SW: SouthWest; W: Wales; WM: West Midlands; YH: Yorks & Humberside.

[3] Weaker (less significant) spatial weights are shown in dashed lines and dotted lines, denoting significance at only 5% and 10% level respectively; red lines denote negative spatial weights.

Importantly, the above method assumes a nonempty set of other cross section units, correlated with the units under consideration, but which may change over time, expand or even vanish. Specifically, motivated by the system GMM approach (Arellano and Bond, 1991; Blundell and Bond (1998), they use these additional cross section (or spatial) units to constitute instruments, in addition to temporal lags normally available as instruments in a panel data setting. Bhattacharjee and Holly (2008a) also develop useful extensions of their methodology to a model with interval censored responses.

In a simple way, the above methodology exploits the panel nature of the data as well as spatial interactions to obtain robust inferences in the presence of potential endogeneity. This is particularly important in microeconomic and spatial contexts where the positions of economic agents or regions in geographical and quality space are determined strategically, and therefore endogenously, as a result of repeated cross section interactions; see also Pinkse et al. (2002) and Conley and Topa (2003).

Similarly, recent developments in the economics of networks suggest that the pattern of connections between individual rational agents shapes their actions and determines their rewards (Goyal, 2007). Understanding, empirically, the precise form of interaction actually observed is an important counterpart to the development of the theory of networks. Thus, in the context of committees and social networks, where theory suggests certain simple equilibrium network structures, these methods can be employed to examine alternative network architectures as well as constraints on information sharing and incentive compatibility.

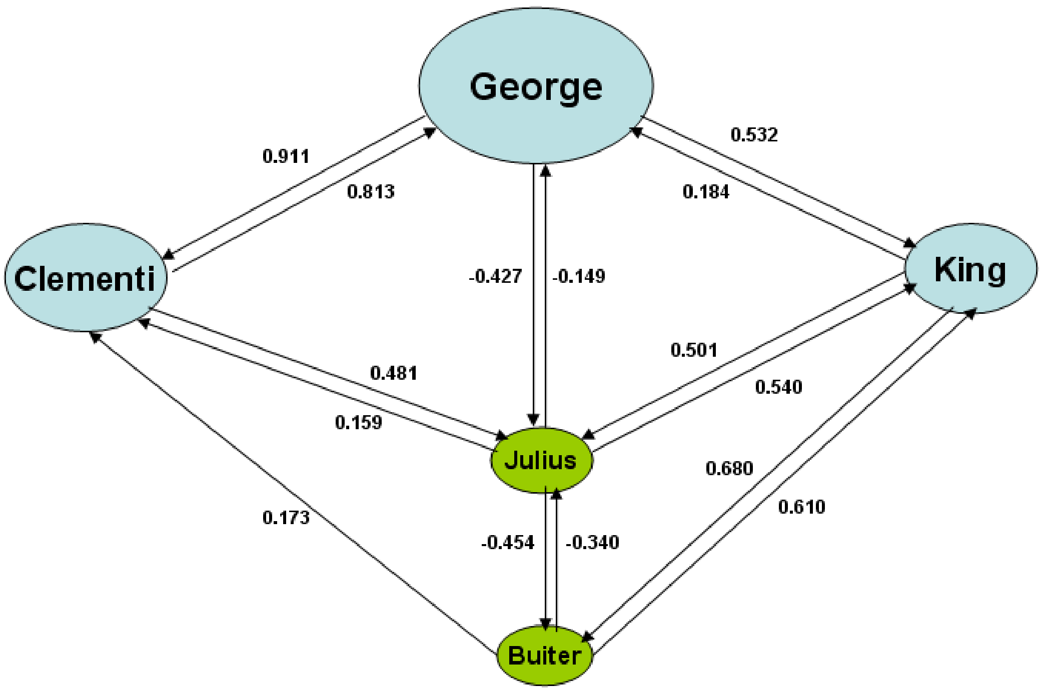

Figure 2: Connections weights within the Bank of England's MPC[4]

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[4] Significant directed interactions are shown in arrows, with corresponding figures relating to estimated connections weights. The ellipses are drawn proportional to total influence of each member in decision making within the committee. Blue ellipses correspond to internal members of the committee, and

One of the important examples is a monetary policy committee (for example, in the European Central Bank or the Bank of England) where members are expected to hold either partisan views depending on the constituency that they represent (such as regions in Europe or sectors in the economy) or otherwise have incentives to behave strategically. Bhattacharjee and Holly (2008a,b) study committee decision making within the Bank of England's monetary policy committee (MPC). Personalities of committee members are reflected in heterogeneity in the policy reaction functions for the different members, as well as in interactions among members that can be strategic or just a result of likemindedness. Evidence points to strong interactions between members, and the network architectures that emerge offer interesting insights into capacity constraints, transaction costs and incentives. Specifically, estimated connections weights for voting behaviour of selected members in the Bank of England's Monetary Policy Committee (Figure 2) provide strong evidences of asymmetry, incomplete connectivity as well as substantial heterogeneity in influence within the committee. Further, negative interaction weights point to strategic voting.

It is an empirical question as to which sets of identifying assumptions, structural restrictions as in Bhattacharjee and Jensen-Butler (2005) or moment restrictions as in Bhattacharjee and Holly (2008a), is more appropriate in the context of any specific application.

4. Spatial interactions driven by unobserved common factors

The above two methodologies are based on an implicit assumption that spatial interactions are structural in a deep sense, rather than being driven by common unobserved factors. In the context of the housing demand example, this implies that whatever factors drive spatial autocorrelation, whether geographic distance or something more intricate, it has no direct effect on regional housing demand. It shapes a particular pattern of spatial interactions, which then affects the spatial diffusion of shocks, and which in turn we can identify and infer upon using the above methods.

However, spatial effects are also potentially driven by common unobserved factors. There is some discussion of this issue in the literature on regional growth and convergence literature; see, for example, Evans and Karras (1996). The above methods are not applicable in the presence of such common factors which affect spatial responses across multiple regions.

Holly et al. (2008) have developed alternative methodology for inferring on spatial interactions in such cases, based on the common correlated effects (CCE) methodology developed in Pesaran (2006). Specifically, they show how these common factors can be adjusted for, in a simple way, by including in the estimated regression model cross section averages of all included regressors and the dependent variable.

Applying their methods to analysis of changes in real house prices within the UK economy at the regional level, Holly et al. (2008) find evidence that adjustment to shocks involves both a region specific and a spatial effect.

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green ellipses to externally appointed members; (Eddie) George was the Governor over the period under analysis.

Shocks to a dominant region - London - are propagated contemporaneously and spatially to other regions, but they impact on other regions with a delay. There are also potential lagged feedback effects back to the dominant region. In turn, London is linked to international developments through stock and bond markets. New York house prices have a direct but lagged effect on London house prices.
Finally, in the context of the methodology used in Holly et al. (2008), it is potentially possible to additionally infer on the drivers of the structural spatial effects. This can be achieved by combining the common correlated effects method with the methods discussed in the previous sections.

5. Sampling of spatial units[5]

The sampling issues relating to the line of enquiry pursued in this paper are important both for credible data analyses as well as for effective collection of spatial data by statistical agencies. It is well understood that survey design plays an important role in data collection, and potentially impacts upon analytical results through sample selection biases.

At the same time, the issues relating to development of effective spatial sampling plans for analyses of the type presented here are not very clearly understood. Therefore, research in this area is of immense importance. To the best of our understanding, there are some important issues which hold potential for important and useful research in the area.

First, the methods considered in this paper explicitly account for endogeneity in the selection of cross section and spatial units. This is in line with prior research, for example by Pinkse et al. (2002), Conley and Topa (2003) and Pesaran (2006). Indeed, it is likely that statistical data collection in spatial (and perhaps even cross section) context will be endogenously related to the size and importance of spatial units, such as towns and regions. Therefore, treating spatial effects as potentially endogenous is an important aspect of such analyses. In doing so, it is expected that part of the adverse impact of sample selection bias is potentially eliminated.

However, and secondly, the effect of such selection biases and potential ways to account for selective sampling have not been adequately studied in the literature. We do not know what the results of such enquiry will be, and point this out essentially as an important direction of future research. Specifically, developing appropriate selection models in a spatial and cross section setting will be an important objective of such research.

Third, such selection models can in turn be effectively used to classify core components of the spatial model from those in the periphery. Importantly, this can then inform statistical agencies regarding further development of effective survey designs for spatial data. Further, this is important for effective identification of moment conditions (essentially based on peripheral units) in some of the reviewed methods such as the one proposed in Bhattacharjee and Holly (2008a).

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[5] The analysis of sampling issues in the paper enriches our discussion, and owes much to the useful recommendation of an anonymous member of the NTTS Scientific Committee.

Finally, and perhaps most importantly, the immense potential of methods reviewed here and related future research derives crucially from the panel nature of the data. This is, in particular, important not only for adjustments for unobserved fixed and random effects at the cross section level, but also for accumulating evidences of spatial interaction and diffusion patterns over time. In this context, it is certainly very important for research that sampling of the same spatial units are continued over substantial time to render such methods effective and useful.

6. Conclusion

In this paper, we reviewed several new methods for drawing inferences on spatial and cross section interactions. In each of the examples considered above, the interactions can be represented in the form of regression models where attributes of an observation unit are a function of endogenous attributes of the other units. Much of the literature on cross section dependence has focused mainly on estimation of the regression coefficients in an underlying model, in a way that is consistent under unrestricted cross section dependence. However, estimation and inferences on the magnitude and strength of spillovers and interactions has been largely ignored. At the same time, as the above examples illustrate, such inferences are important, not least because they have structural interpretations and provide useful interpretation and structural explanation for the strength of any interactions.

Research in the above area, both empirical and econometric, is still in its infancy. Here, we have discussed some new methodologies and illustrated these with some examples. We have also discussed limitations of the available methods. Most importantly, our survey suggests several new ideas for further and useful research. This includes development of new methodologies in combining the common factor approach with structural spatial interactions, but also important new research ideas relating to sampling of spatial units.

Also, further applications of the methods need to be developed. For example, an important application area would be economic convergence of countries and regions. Previous research suggests that there are stable differences in productivity across regions in the EU. Potentially, such spatial inequality can be explained by technology transfer, where some regions are more efficient in generating new technology or technology absorption. In turn, technology transfer is often related to trade (imports and/ or exports), FDI etc., while technology absorption depends on human capital, R&D and similar features. However, theory provides no clear guidance as to which of these channels are more important, or indeed if relative importance varies across regions. Further, while some of the diffusion can be explained by the pull of common factors, there may be institutional features that enhance or depress technology transfer between specific pairs of regions. In empirical studies, it is useful to allow for technology transfer in relatively unrestricted manner and further, to infer on the strength and direction of inter-region diffusion while being agnostic about the specific drivers of such interaction; see Bhattacharjee and Jensen-Butler (2005) for a simulation based on the US, and Bhattacharjee et al. (2007) for an application to Danish regions. Further research along similar lines will be important for our understanding of convergence both within the EU and elsewhere.

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